Connecting theory with experiment: from gluons to g-2

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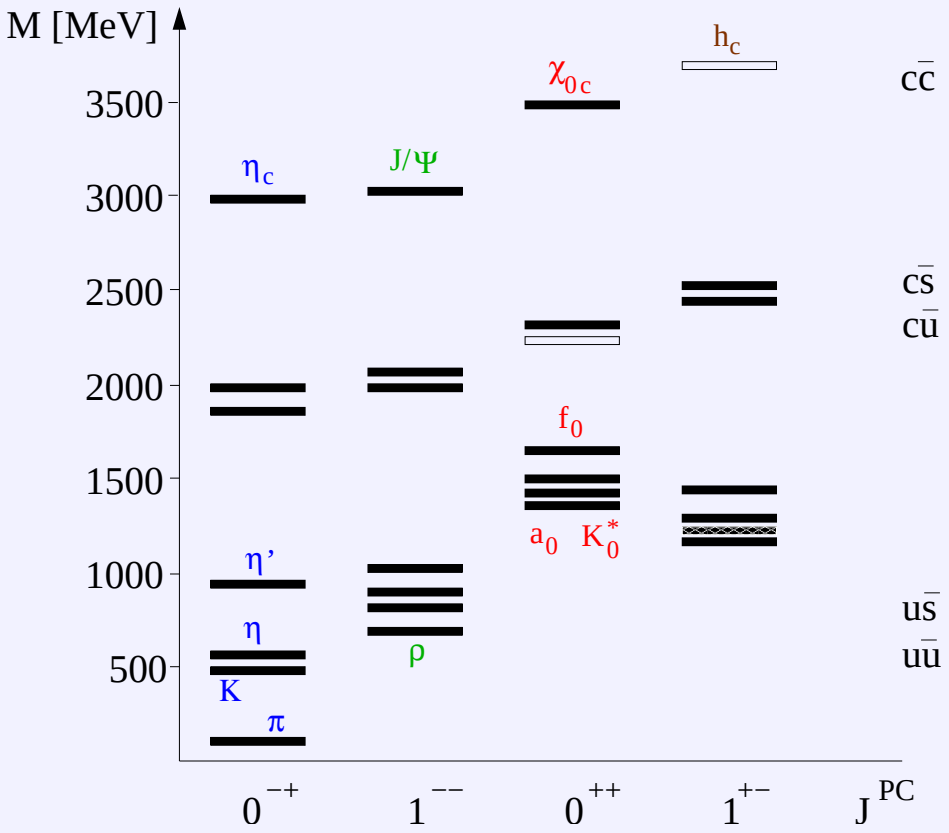
T. Goecke, C.F. and R. Williams, in preparation
C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408
R. Alkofer, C.F. and R. Williams, Eur. Phys. J. A **38**, 53 (2008)

Spectrum of mesons

Experiment:
(Sketch)



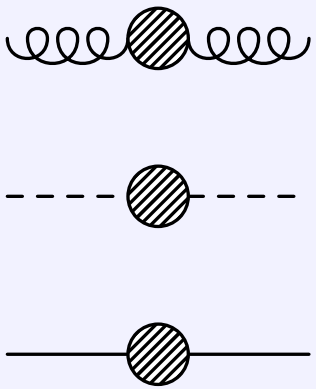
Goldstone-Bosons $\leftrightarrow$ Bound pseudoscalar
Quark-Antiquark-states: $\pi^{\pm,0}, K^{\pm,0,\bar{0}}, \eta$

QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ - \int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Propagators in momentum space:



$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

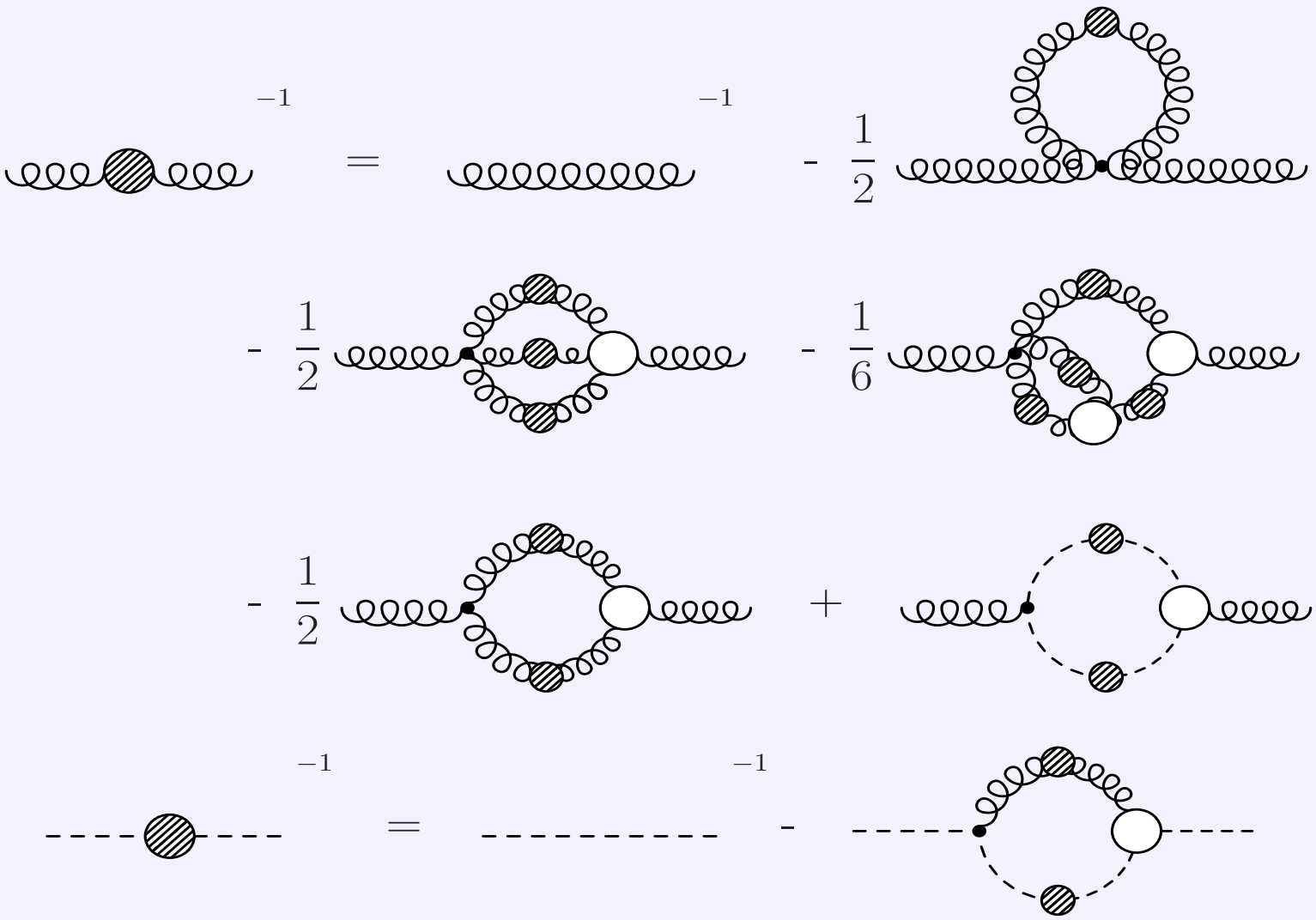
$$D^{\text{Ghost}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

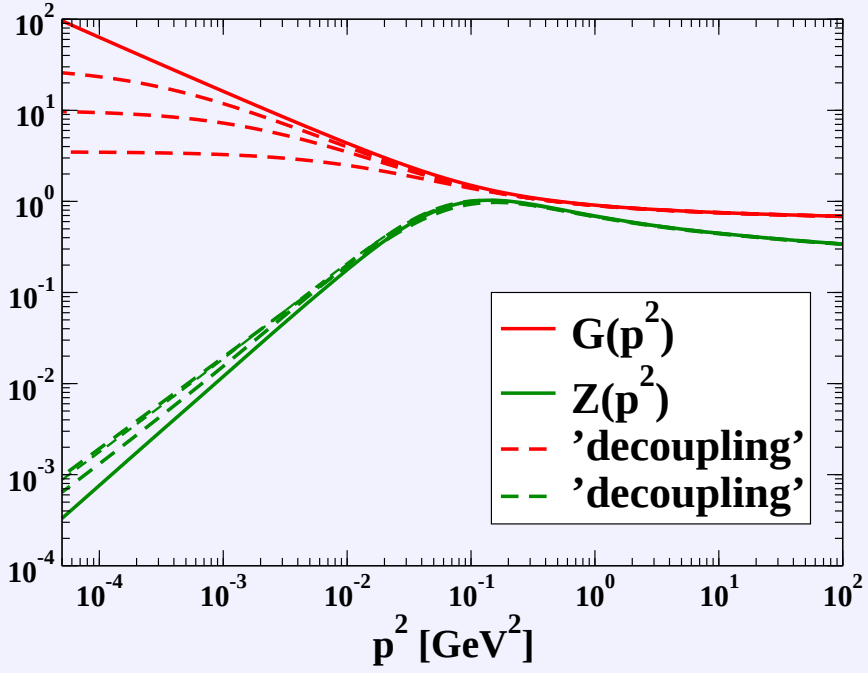
Lattice vs. DSE/FRG/BSE: Complementary!

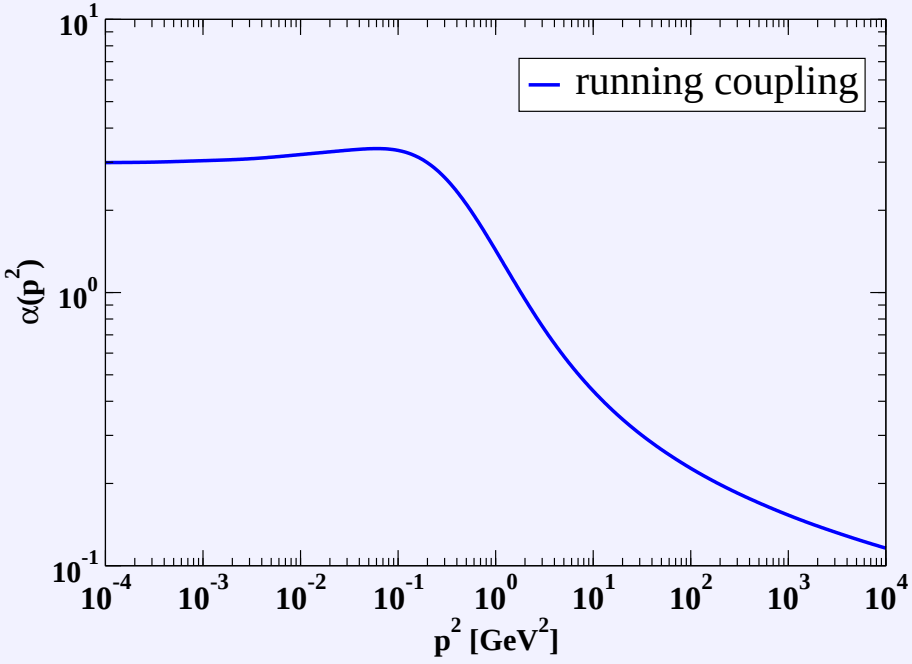
- Lattice simulations
  - Ab initio
  - Gauge invariant
- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  Bethe-Salpeter-equations (BSE)
  - Analytic solutions at small momenta
  - Chiral symmetry: light quarks and mesons
  - Space-Time-Continuum
  - Accommodation of very different scales possible
  - Chemical potential: no sign problem

Dyson-Schwinger equations (DSEs)



Ghost, Glue and Coupling





- dynamically generated scale
- fixed point of coupling $\alpha(p^2) = g^2/(4\pi)Z(p^2)G^2(p^2) \approx 9/N_c$
- deep infrared ($p < 50$ MeV): scaling vs. decoupling

CF and Alkofer, PLB 536 (2002) 177.

C. Lerche and L. von Smekal, PRD **65**, 125006 (2002).

C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408.

Infrared Structure of YM-theory: $p^2 << \Lambda_{QCD}$

Two type of **analytic and exact** solutions for complete tower of DSEs:

Scaling:

n ghost, *m* gluon legs

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

- $G(0) = \infty$
- $\kappa > 0$
- Kugo Ojima confinement scenario supported!

Decoupling:

n ghost, *m* gluon legs

$\Gamma^{0,2}(p^2) \sim 1/(p^2)$, others finite

- $G(0) = \textit{finite}$
- gluon 'mass' $(p^2\ \Gamma^{0,2})_{p^2=0}$ not determined
- BRST symmetry broken

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

C.F. and J. M. Pawlowski, PRD **75** (2007) 025012; PRD **80**, 025023 (2009)

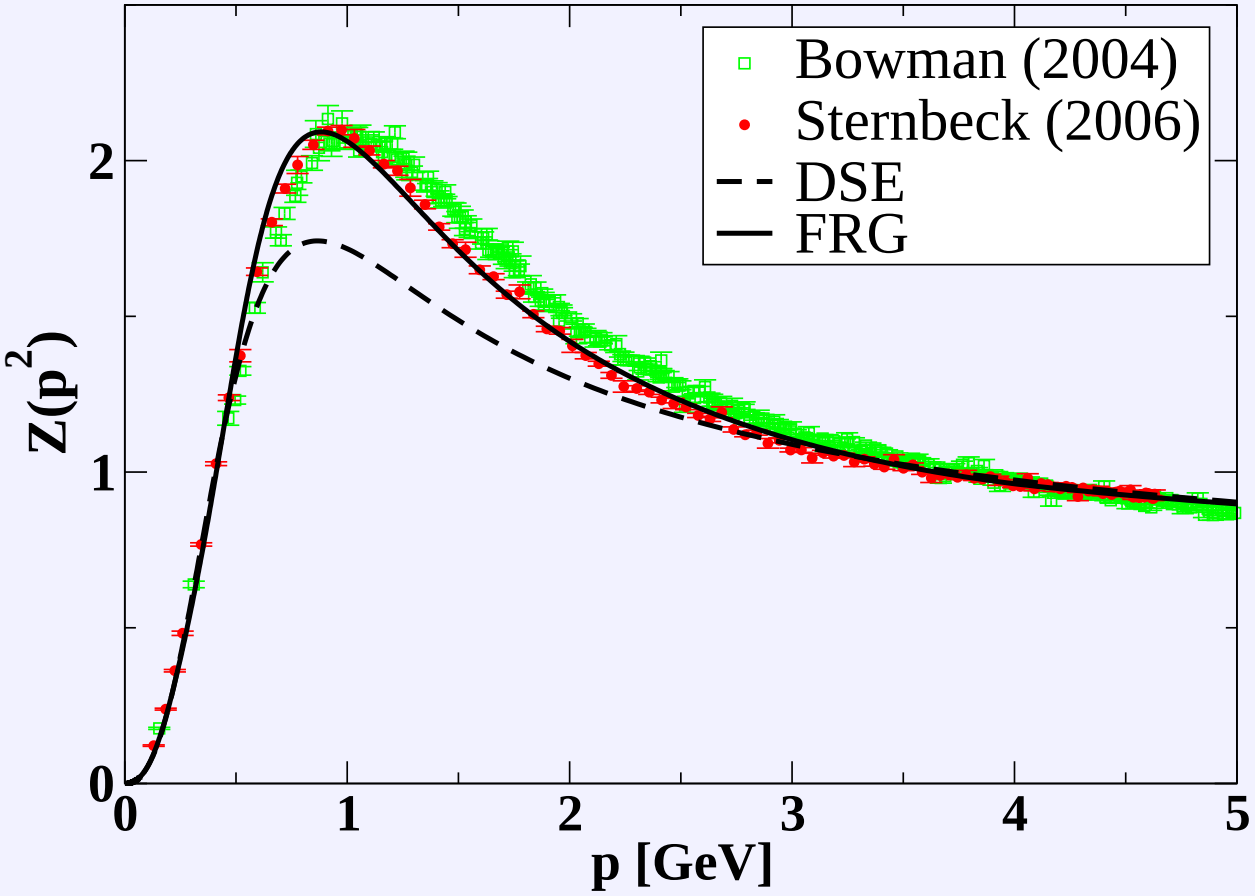
Boucaud, Leroy, Yaouanc, Micheli, Pene, Rodriguez-Quintero, JHEP **0806** (2008) 012.

Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008).

C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408.

A. Maas, Phys.Lett.B689, 107-111 (2010)

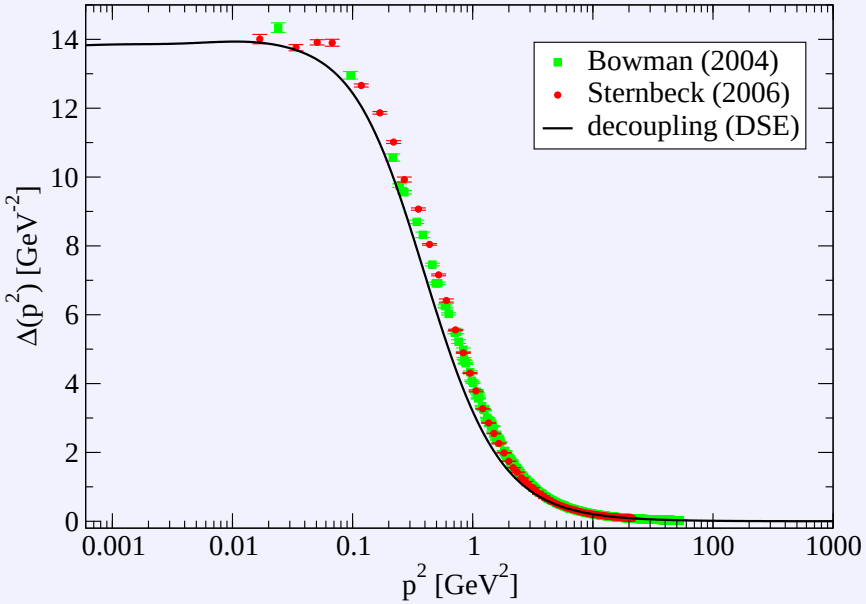
DSEs vs Lattice I



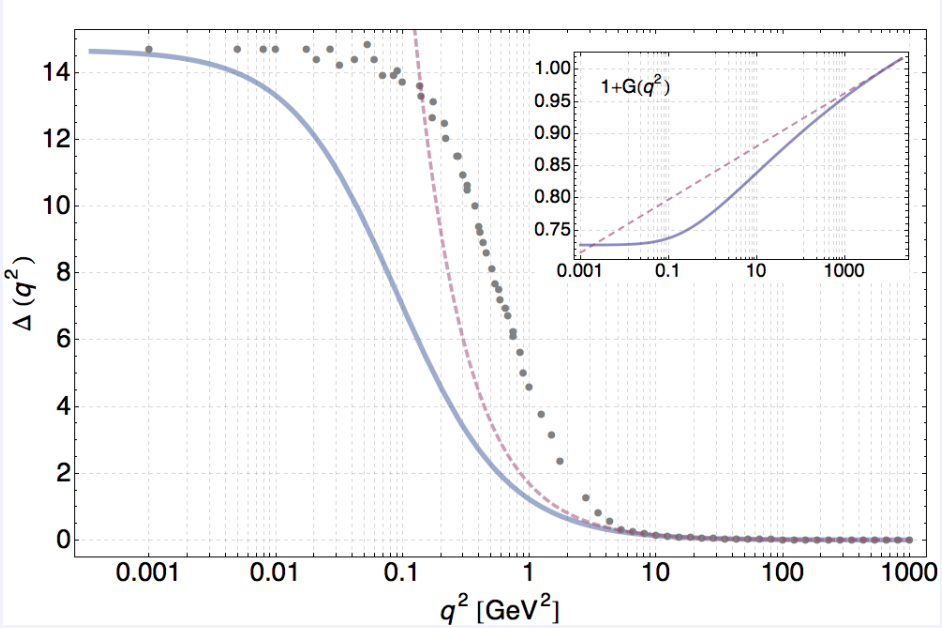
- DSE vs FRG: Effect of four-gluon-interaction
- Deep infrared: Interesting and subtle questions

C.F., A. Maas and J. M. Pawlowski, Annals Phys. **324** (2009) 2408.

DSE vs Lattice II



C.F., Maas and Pawlowski, Ann. Phys. **324** (2009) 2408.

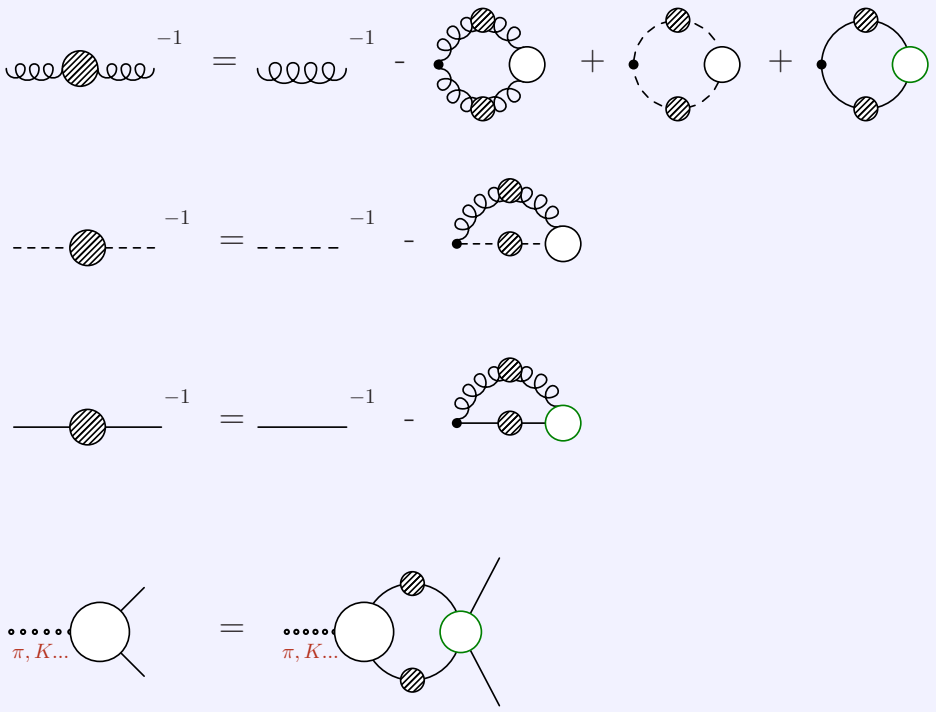


Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008).

- Mid-momentum behaviour crucial for phenomenology!
- One-parameter scaling-decoupling family also seen on the lattice

A. Maas, Phys.Lett.B689, 107-111 (2010)

From gluons and quarks to mesons



- Central quantity: quark-gluon vertex
- Meson structure beyond rainbow-ladder
  Alkofer, C.F., Llanes-Estrada, Schwenzer, Annals Phys.324:106-172,2009.
  C.F. and R. Williams, PRD **78**, 074006 (2008).
  C.F. and R. Williams, PRL **103** (2009) 122001.
  $\rightarrow$ **Richard Williams**
- Baryon structure
  $\rightarrow$ **Gernot Eichmann (6C)**
- $U_A(1)$-problem
- $\pi\gamma\gamma$ and g-2

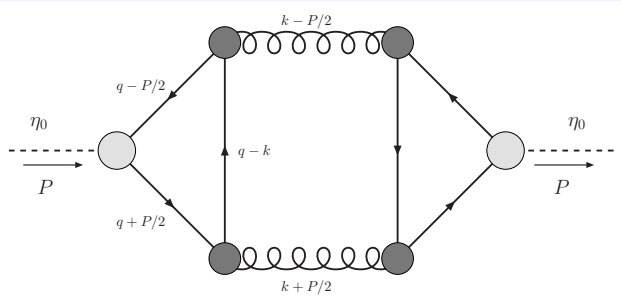
The $U_A(1)$-problem

η' too heavy for a Goldstone boson $\longrightarrow m^{\chi}_{\eta'} \neq 0$ in chiral limit

- Conjecture: Coloured infrared singularities generate $m^{\chi}_{\eta'} \neq 0$

  J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

- $\Gamma^{quark-gluon}(p^2) \sim (p^2)^{-1/2-\kappa}$ provides correct IR-strength



- our results:

$m^{\chi}_{\eta'}$ [MeV]	*Top.susc.* [MeV^4]	θ	m_η [MeV]	$m_{\eta'}$ [MeV]
748	169	-23.2	479	906

R. Alkofer, C.F. and R. Williams, Eur. Phys. J. A **38**, 53 (2008)

Hadronic contributions to g-2 I

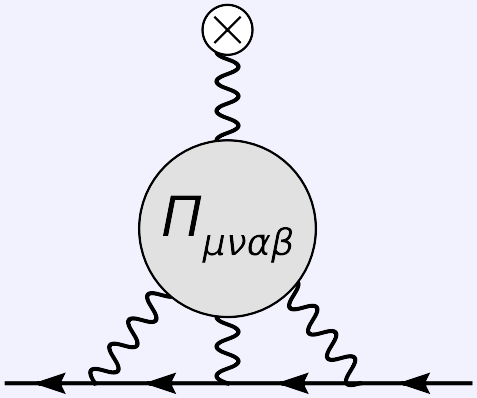
Experiment: $11\,659\,208.0(6.3) \times 10^{-10}$

G. W. Benett *et al.* (Muon g-2 Collaboration), PRD **73**, 072003 (2006)

Theory: $11\,659\,179.0(6.5) \times 10^{-10}$

F. Jegerlehner and A. Nyffeler, Phys. Rept. **477**, 1 (2009)

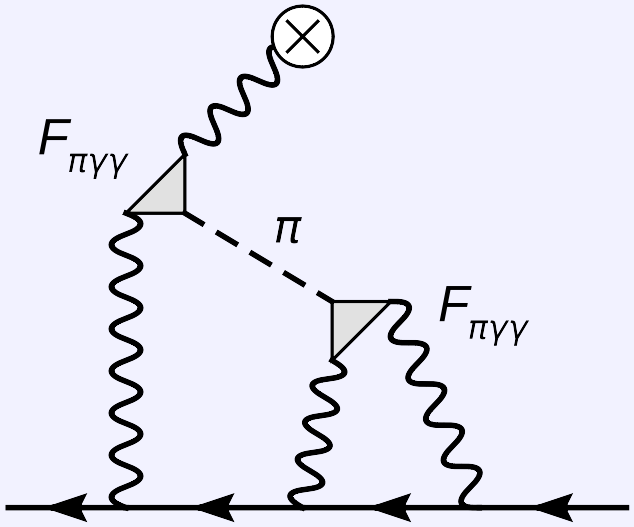
Problem: 'lbl-contribution' with $11.6(3.9) \times 10^{-10}$:

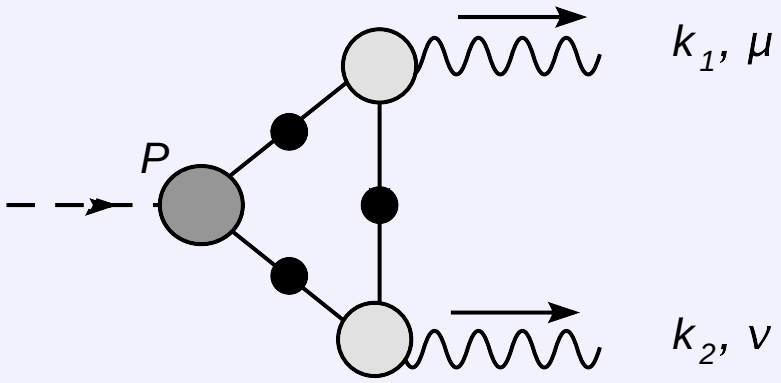


- Cannot be determined from experimental input
- Multi-Scale problem!
- $\rightarrow$ NJL-model, effective theory (RLA), VMD

Goal: 'Ab initio' calculation of lbl contribution

Hadronic contributions to g-2 II



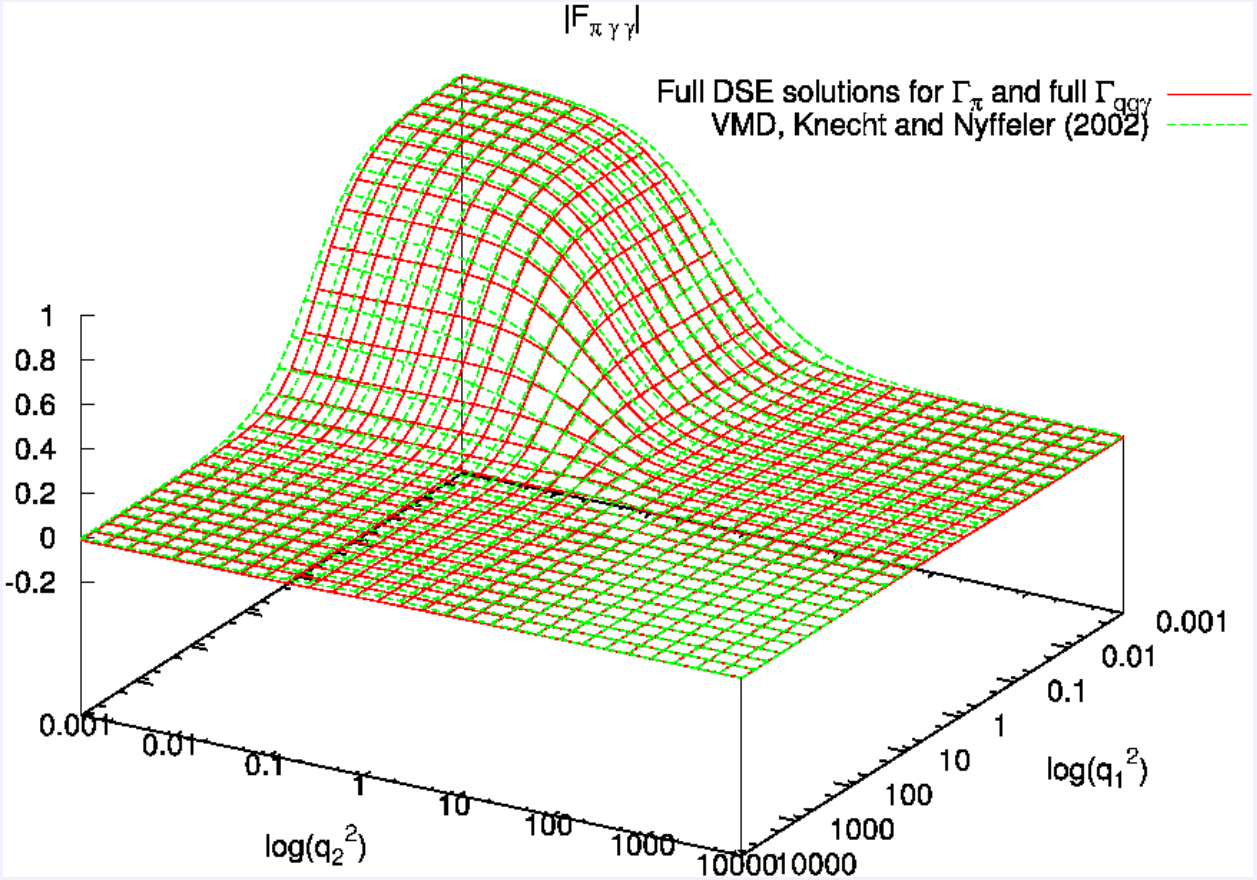


- leading contribution to lbl: pion exchange
- need to determine $\pi\gamma\gamma$ 'off-shell form factor'
- use DSE for fermion-photon vertex

First step:

- Maris-Tandy model $\rightarrow$ **Peter Tandy (plenary)**

Preliminary results: $\pi\gamma\gamma$



- correct asymptotics
- differences in soft photon regions

Preliminary results: anomalous magnetic moment

Preliminary results:

$\Gamma_{qq\gamma}$	Γ_π	off shell	$a_\mu^{(\pi^0)}[10^{-10}]$
simple	simple	$\times$	8.3
simple	simple	$\surd$	7.1
full	simple	$\surd$	5.9
full	full	$\times$	5.4
full	full	$\surd$	5.0

T. Goecke, C.F. and R. Williams, in preparation

to be compared with

$a_\mu^{(\pi^0)}[10^{-10}] = 6.5(2)$ A. E. Dorokhov and W. Broniowski, Phys. Rev. D **78**, 073011 (2008)

$a_\mu^{(\pi^0)}[10^{-10}] = 7.2(12)$ A. Nyffeler, Phys. Rev. D **79** (2009) 073012

Summary

Gluon sector of QCD:

- Gluon propagator: Quantitative agreement with lattice results

Light mesons:

- topological mass contribution to η, η'.
- hadronic light-by-light scattering in g-2: first results available

Confinement:

- Positivity violations in gluon propagator

  R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014
- Zero Polyakov-Loop below T_c (quenched)

  C.F., Maas, Mueller, EPJC in press, arXiv:1003.1960
- Chiral and Deconfinement phase transition

  C.F., Maas, Mueller, EPJC in press, arXiv:1003.1960

  C.F., PRL **103** (2009) 052003

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TECHNISCHE UNIVERSITÄT DARMSTADT

HELMHOLTZ | GEMEINSCHAFT

GSI

HIC for FAIR

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

EMMI

Helmholtz-Alliance: Extremes of density and temperature; cosmic matter in the laboratory